A dynamic field account to language-related brain potentials

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Summary. We present a phenomenological modeling account to event-related brain potentials (ERP) in syntactic language processing. For a paradigmatic ERP experiment on the processing of phrase structure violations in German [Hahne and Friederici (1999). Electrophysiological evidence for two steps in syntactic analysis: Early automatic and late controlled processes. *Journal of Cognitive Neuroscience*, 11(2):194 – 205] we derive a context-free grammar representation of the stimulus material. We describe the phrase structure violation by a prediction error in an interactive left-corner parser that is mapped onto a dynamic field by means of dynamic cognitive modeling (DCM). Our model phenomenologically replicates the experimentally observed P600 ERP component elicited by the violation.

1 Introduction

Event-related brain potentials (ERP) are an important online measure in the cognitive neurosciences (Handy 2005, Kutas and van Petten 1994). Conventionally, ERPs are obtained from averaging stimulus-locked electroencephalograms (EEG) across trials. They exhibit characteristic topographies of positive and negative voltage deflections in comparison to a baseline condition at particular latency times. ERPs are commonly classified according to their polarity and latency, such that "N400" denotes a negative peak 400 ms after

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stimulus presentation and "P600" refers to a positive peak 600 ms after stimulus. The conventional ERP averaging analysis presupposes the existence of an invariant "ERP signal" that is concealed by the spontaneous EEG regarded as stationary and ergodic noise (Regan 1972, beim Graben et al. 2000). However, other interpretations assume stimulus-induced phase resetting (Makeig et al. 2002; 2004) or stimulus-induced amplitude modulation (Nikulin et al. 2007) (see also Becker et al. (2008) for a critical comparison of these models). From a dynamical system point of view, Başar (1980; 1998) and beim Graben et al. (2000) interpreted single-trial ERP time series as images of phase space trajectories originating from randomly distributed initial conditions under an appropriate observation model. Compatible with this picture is also the suggestion of Hutt et al. (2000) and Hutt (2004) that ERP components correspond to saddle nodes forming (stable) heteroclinic sequences (SHS: Afraimovich et al. (2004), Rabinovich et al. (2008)). In this interpretation, ERP phase space trajectories would be confined to a stable heteroclinic channel (SHC: Afraimovich et al. (2004), Rabinovich et al. (2008)) forming a kind of "bottleneck" (beim Graben et al. 2000).

In the language domain, ERPs found several applications. The so-called mismatch negativity (MMN) reflects phonological and lexical mismatches (Näätänen et al. 1997, Dehaene-Lambertz 1997, Rugg 1984, Pulvermüller et al. 1995). Whereas the abovementioned N400 and P600 are sensitive for semantic (Kutas and Hillyard 1980; 1984, Kolk et al. 2003, Herten et al. 2005), syntactic (Osterhout and Holcomb 1992, Hagoort et al. 1993)) and pragmatic (Noveck and Posada 2003, Nieuwland and Kuperberg 2008, Drenhaus et al. 2006)) processing problems.

Neurodynamical models of language-related brain potentials have to take one crucial characteristics of language into account: Language is categorial and symbolic. In a recent publication, Kiebel et al. (2009) employed SHS dynamics for phonetic speech recognition in order to model the observed mismatch negativity (Näätänen et al. 1997, Dehaene-Lambertz 1997). The same ERP component observed in word recognition experiments (Rugg 1984, Pulvermüller et al. 1995), has been tackled by Garagnani et al. (2007; 2008). In the syntactic domain, Hagoort (2003; 2005) described the P600 ERP component through unification costs in the unification model of Vosse and Kempen (2000; 2009). Using different kinds of formal grammars, beim Graben et al. (2008a) and Gerth and beim Graben (2009) made first attempts towards dynamical systems model of syntactic reanalysis within the framework of dynamic cognitive modeling (DCM: beim Graben and Potthast (2009)).

In this chapter, we demonstrate the basic steps of DCM by means of a paradigmatic ERP experiment (Hahne and Friederici 1999). We employ functional representations of complex linguistic data structures (beim Graben et al. 2008b) in order to provide a phenomenological account to the observed syntactic ERP components.

2 ERP-Experiment

In an ERP experiment in German, Hahne and Friederici (1999) presented three types of sentences visually word-by-word to their subjects. The conditions are illustrated by means of the following examples.

(1) Die Gans wurde **gebraten**.
The goose was **grilled**.
"The goose was grilled."

(2) *Die Gans wurde im **gebraten**.
The goose was in the **grilled**.
"The goose was grilled in the."

(3) Die Gans wurde im Ofen **gebraten**.
The goose was in the oven **grilled**.
"The goose was grilled in the oven."

Sentences of type (1) are grammatically well-formed. They served as the controlling baseline condition. By contrast, sentences of type (2) exhibit a *phrase structure violation* (indicated by the asterisk) that is illustrated by the filler sentences of type (3): here, a prepositional phrase *im Ofen* ("in the oven") modifies the main verb *gebraten* ("grilled"). A prepositional phrase is introduced by a preposition, such as *im* ("in the") that requires an adjacent noun, here *Ofen*. This noun is omitted in type (2), thereby leading to the phrase structure violation that becomes manifest with the concluding, critical, word *gebraten*, printed in bold font in the examples above. Measuring event-related brain potentials, elicited by the critical word entails a P600 component broadly distributed over parietal recording sites. Figure 1 displays the P600 in the grand ERP average.

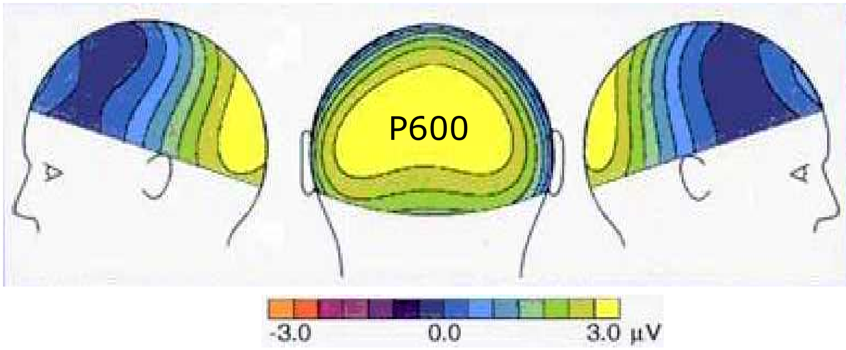


Fig. 1. The P600 ERP component elicited by phrase structure violation condition (2) in comparison to the correct condition (1) in the experiment by Hahne and Friederici (1999) (20% contrast). Shown are spherical splines of difference waves (2) – (1). Modified reprint with permission of MIT Press.

The P600 ERP is robustly associated to many kinds of ungrammaticalites (Osterhout and Holcomb 1992, Hagoort et al. 1993), syntactic ambiguities (Osterhout et al. 1994, beim Graben et al. 2000, Frisch et al. 2002), and syntactic complexity (Kaan et al. 2000, Kaan and Swaab 2003). In the context of the experiment of Hahne and Friederici (1999), it can be related to the violation of a particular expectancy generated by the human language processor: Encountering the preposition *im* predicts the occurrence of a noun, such as *Ofen* as the next input word. In the next section, we present a computational model for this predictive dynamics.

3 Dynamic Cognitive Modeling

According to the "computer metaphor of the mind" (Pylyshyn 1986), cognition is essentially time-discrete symbol manipulation obeying combinatorial rules. In contrast, the brain as a nonlinear dynamical system operates in continuous time with continuous activation patterns instead of discrete symbolic representations. In order to bridge the gap between computational psycholinguistics on the one hand and computational neuroscience on the other hand, thereby constituting a new discipline of *computational neurolinguistics*, two different mapping problems have to be solved. First, discrete mental symbolic representations have to be mapped onto continuous activation patterns in neurodynamical systems. This mapping can be achieved through filler/role decompositions and tensor product representations (Mizraji 1989, Smolensky 1990, Smolensky and Legendre 2006, Smolensky 2006). Second, these activation patterns have to be connected through trajectories in continuous time.

This embedding can be straightforwardly established by means of winnerless competition and heteroclinic sequences (Rabinovich et al. 2001, Afraimovich et al. 2004, Rabinovich et al. 2008).

Taken together, tensor product representations and winnerless sequential dynamics constitute dynamic cognitive modeling (DCM: beim Graben and Potthast (2009)) as a three tier top-down approach comprising the levels of (1) cognitive processes; (2) their state space representations; and (3) their neurodynamical realizations. In the following subsections, we illustrate DCM on the basis of the language-processing ERP experiment from Sec. 2.

3.1 Data structures and algorithms

Since DCM is a top-down approach, we first have to find an appropriate linguistic representation of the sentence examples (1) – (3). Sentences such as the filler (3) are hierarchically organized symbolic data structures: A sentence[2] `S` consists of a subject, realized as a first *noun phrase* $\texttt{NP}_1$ *die Gans* ("the goose") and a predicate, called *verbal phrase* `VP'`. In our example (3) the predicate is headed by an auxiliary `Aux` *wurde* ("was") complemented by another verbal phrase `VP`. This phrase in turn consists of the verb `V` *gebraten* ("grilled") modified by the prepositional phrase `PP`. Finally, the prepositional phrase is headed by the preposition `P` *im* ("in the") with noun phrase complement $\texttt{NP}_2$ Ofen ("oven"). A suitable way to display such syntactic dependencies are *phrase structure trees*. Figure 2 shows the phrase structure tree of sentence (3) reflecting our hierarchical analysis.

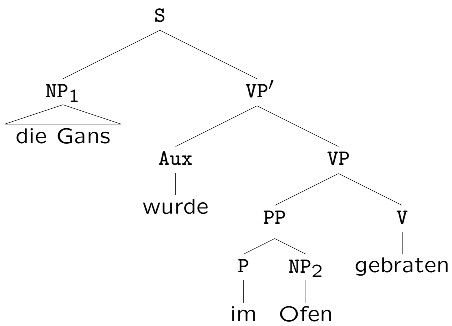


Fig. 2. Linguistic phrase structure tree of filler sentence (3). Syntactic categories are `S`: sentence, $\texttt{NP}_1$, $\texttt{NP}_2$: noun phrases, `VP'`, `VP`: verbal phrases, `Aux`: auxiliary, `V`: verb, `PP`: prepositional phrase, `P`: preposition.

[2] As usual in computer science, we write symbols in `typewriter` font here.

In the next step, we create a context-free grammar (CFG) (Hopcroft and Ullman 1979) from the tree in Fig. 2, after discarding the particular lexical material because that varied across the different trials in the ERP study. A CFG is a collection of rules $A \to \gamma$ where $A \in \mathbf{N}$ is a syntactic category (a nonterminal) and $\gamma \in (\mathbf{T} \cup \mathbf{N})^*$ is a string formed of terminal or nonterminal symbols. As terminals we regard the symbols from the set $\mathbf{T} = \{\mathtt{NP}_1, \mathtt{NP}_2, \mathtt{Aux}, \mathtt{V}, \mathtt{P}\}$, while the remaining categories $\mathbf{N} = \{\mathtt{S}, \mathtt{VP}', \mathtt{VP}, \mathtt{PP}\}$ are considered to be nonterminals. Since every branching in the tree corresponds to a production in the CFG, we obtain the following CFG:

$$
\begin{aligned}
\mathtt{S} &\to \mathtt{NP}_1 \quad \mathtt{VP}' \\
\mathtt{VP}' &\to \mathtt{Aux} \quad \mathtt{VP} \\
\mathtt{VP} &\to \mathtt{PP} \quad \mathtt{V} \\
\mathtt{PP} &\to \mathtt{P} \quad \mathtt{NP}_2 \, .
\end{aligned} \tag{4}
$$

In terms of phrase structure trees and context-free grammars the human language processor can be most appropriately described as an interactive (Wegner 1998, beim Graben et al. 2008a) left-corner parser (Aho and Ullman 1972, Demers 1977, Hale 2011). Interactivity refers to the fact that the parser is permanently perturbed by new incoming input words. In a left-corner architecture, the parser takes new input as evidence for making predictions about subsequent words. Then, ungrammatical input leads to prediction failure, that becomes reflected by the P600 ERP observed in experiments such as in our example (Hahne and Friederici 1999).

For the sake of simplicity, we refrain from elaborating a complete left-corner parser here. Instead, we present the required cognitive architecture as an algebraic representation π (beim Graben et al. 2008a, beim Graben and Potthast 2009) of the terminal alphabet $\mathbf{T}$ on the space of phrase structure trees P, i.e., for every input word $w \in \mathbf{T}$, $\pi(w) : P \to P$ maps a tree $p \in P$ to another tree $p' = [\pi(w)](p) \in P$.

In order to process example sentence (3), we initialize the parser with a "vacuum state" s_1, the empty tree[3] $\emptyset \in P$. Scanning the first noun phrase $\mathtt{NP}_1$ *die Gans* ("the goose") from the environment yields the second state $s_2 = [\pi(\mathtt{NP}_1)](s_1)$, which is a tree where $\mathtt{NP}_1$ is the confirmed left corner as a result from expanding the first rule $\mathtt{S} \to \mathtt{NP}_1 \quad \mathtt{VP}'$ of CFG (4). In contrast, the right corner is being predicted, indicated by brackets, $[\mathtt{VP}']$ (Hale 2011).

[3] Note that a tree in graph theoretical sense is a set of nodes K connected through edges $E \subset K \times K$. Therefore, the empty tree corresponds to the empty set of nodes $K = \emptyset$.

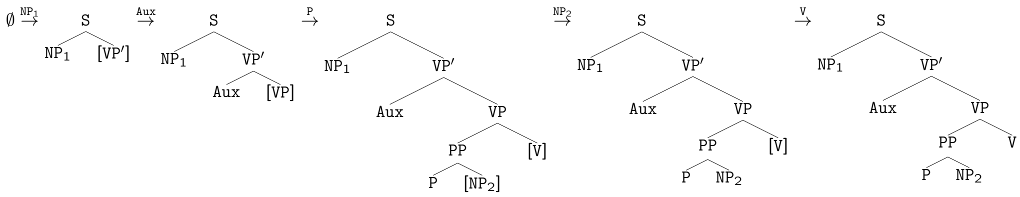


Fig. 3. Interactive left-corner parse of example sentence (3) according to CFG (4).

In the third step the parser scans the auxiliary `Aux` *wurde* ("was") from the input, thereby confirming the previous prediction of a verbal phrase $\texttt{VP}'$, yielding tree $s_3 = [\pi(\texttt{Aux})](s_2)$. According to rule 2 of CFG (4), another verbal phrase [`VP`] becomes then predicted. The processor proceeds with the preposition `P` *im* ("in the") in the fourth step, confirming the predicted `VP`, by making two further predictions of [$\texttt{NP}_2$] and [`V`]. Scanning the noun phrase $\texttt{NP}_2$ Ofen ("oven") in step five, confirms $\texttt{NP}_2$. In the final step, the scanned verb `V` *gebraten* ("grilled") eventually confirms the predicted `V`, retaining the phrase structure tree of the wellformed sentence from Fig. 2. Figure 3 shows the complete parse as a sequence $(s_1, s_2, s_3, s_4, s_5, s_6)$ of 6 left-corner trees generated by the input (3).

Applying the left-corner algorithm to the phrase structure violation sentence (2), yields another tree sequence $(s_1, s_2, s_3, s_4, s_7)$ of 5 left-corner trees displayed in Fig. 4.

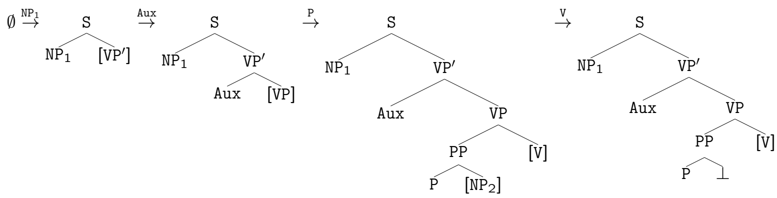


Fig. 4. Interactive left-corner parse of phrase structure violation (2) according to CFG (4). Prediction failure is indicated by $\perp$ ("bottom") and reflected by the P600 ERP (Fig. 1).

Obviously, the first four processing steps are the same as in Fig. 3 because the input is interactively fetched from the environment. However, in step five the scanned verb fails to confirm the predicted noun phrase $\texttt{NP}_2$, indicated by the symbol $\perp$ ("bottom") in the respective tree s_7. At this point, the processing breaks down, which is reflected by the ungrammaticality P600 in the ERP (Fig. 1).

3.2 State space representations

In order to solve the first mapping problem mentioned in Sec. 3, the construction of vector space representations of complex symbolic data structures, we employ a filler/role decomposition of the trees in Figs. 3, 4 (Smolensky 1990, Smolensky and Legendre 2006, Smolensky 2006). To this end, we firstly identify the syntactic categories with a set of *fillers* $F = \mathbf{T} \cup \mathbf{N} \cup \mathbf{N}' \cup \{\perp\} = \{\mathtt{NP}_1, \mathtt{NP}_2, \mathtt{Aux}, \mathtt{V}, \mathtt{P}, \mathtt{S}, \mathtt{VP}', \mathtt{VP}, \mathtt{PP}, [\mathtt{VP}'], [\mathtt{VP}], [\mathtt{PP}], \perp\}$. Note that both, basic categories $\mathbf{N}$ of the grammar (4) and their predicted left-corner counterparts $\mathbf{N}'$ become fillers in this step. Moreover, also the indicator of prediction failure $\perp$ in tree s_7 assumes a filler variable. Secondly, we introduce a set of *roles* $R = \{r_1, r_2, r_3\}$ for the positions in an elementary binary branching tree: r_1 for the mother node, r_2 for the left daughter node and r_3 for the right daughter node (cf. beim Graben et al. (2008a;b), beim Graben and Potthast (2009) for details). A filler/role decomposition for an elementary tree such as s_2 in Fig. 3 is then a set of ordered pairs of fillers *bound* to their respective roles. In our example we obtain $f_2 = \{(\mathtt{S}, r_1), (\mathtt{NP}_1, r_2), ([\mathtt{VP}'], r_3)\}$, as $\mathtt{S}$ occupies the mother node, $\mathtt{NP}_1$ the left daughter, and the predicted $[\mathtt{VP}']$ the right daughter node position. The set f_2 itself is a *complex filler* that could bind to another role in a more complex tree. As an example consider the third step s_3 in the tree sequence in Fig. 3. Here, the complex subtree filler $f_R = \{(\mathtt{VP}', r_1), (\mathtt{Aux}, r_2), ([\mathtt{VP}], r_3)\}$ is bound to the right daughter node of the first-level tree: $f_3 = \{(\mathtt{S}, r_1), (\mathtt{NP}_1, r_2), (f_R, r_3)\}$, or, after evaluation, $f_3 = \{(\mathtt{S}, r_1), (\mathtt{NP}_1, r_2), (\{(\mathtt{VP}', r_1), (\mathtt{Aux}, r_2), ([\mathtt{VP}], r_3)\}, r_3)\}$. In general, the set of complex fillers is recursively defined as

$$\begin{aligned} F_0 &= F \\ F_{n+1} &= \wp(F_n \times R) \\ F_\infty &= \bigcup_{n=0}^{\infty} F_n \ , \end{aligned} \tag{5}$$

where $\wp(\cdot)$ denotes the power set of an argument (beim Graben et al. 2008b).

After having mapped all trees in Figs. 3, 4 to sets of ordered pairs (of sets of ordered pairs, etc) by virtue of the filler/role decomposition (5), we next carry out tensor product representations (Mizraji 1989, Smolensky 1990, Smolensky and Legendre 2006, Smolensky 2006) by mapping the fillers F onto a vector space $\mathcal{V}_F = \psi(F)$ and by mapping the roles R onto another vector space $\mathcal{V}_R = \psi(R)$. Where $\psi : F_\infty \to \mathcal{F}$ additionally obeys

$$\psi(\{(f, r), (f', r')\}) = \psi(f) \otimes \psi(r) \oplus \psi(f') \otimes \psi(r') \ , \tag{6}$$

for fillers $f \in F_n, f' \in F_m$ and roles $r, r' \in R$. Hence, the resulting space $\mathcal{F}$ is the *Fock space*

$$\mathcal{F} = \left(\bigoplus_{n=1}^{\infty} \mathcal{V}_F \otimes \bigotimes_{k=0}^{n} \mathcal{V}_R \right) \oplus \mathcal{V}_R \,, \tag{7}$$

well-known from quantum field theory (Haag 1992, Smolensky and Legendre 2006, Aerts 2009). Here, we choose a particular Fock space representation, namely a function space over the unit sphere, figuring a compact *feature space* in dynamic field theory (Erlhagen and Schöner 2002, Schöner and Thelen 2006).

The unit sphere $S \subset \mathbb{R}^3$ is parameterized through polar coordinates: radius $r \in [0, 1]$, polar angle $\vartheta \in [0, \pi]$, and azimuth $\varphi \in [0, 2\pi[$. In our model we use complex radial oscillations

$$f_i(r) = \mathrm{e}^{ikr+\phi_i} \,, \qquad k = \frac{2\pi \mathrm{i}}{N_F} \,, \tag{8}$$

for the ith filler where $N_F = 10$ is the number of basic categories, including prediction failure $\perp$. The additional phases $\phi_i = 0$ for thus basic categories and $\phi_i = \pi/8$ for left-corner predicted categories render the latter similar to the former (Smolensky and Legendre 2006). For the tree roles R we use spherical harmonics

$$Y_{jm}(\vartheta, \varphi) = \sqrt{\frac{2j+1}{4\pi} \frac{(j-m)!}{(j+m)!}} \, P_{jm}(\cos\vartheta) \, \mathrm{e}^{\mathrm{i}m\varphi} \tag{9}$$

with $j = 1$ and $m \in \{-1, 0, 1\}$ as elaborated in great detail by beim Graben et al. (2008b) and beim Graben and Potthast (2009). Spherical harmonics are particularly well-suited for DCM as they exhibit nice recursion properties, expressed by Clebsch-Gordan expansions, in order to represent deeper tree levels by means of higher order spatial harmonics (the P_{jm} in Eq. (9) are the associated Legendre polynomials). Then, tensor products (6) become pointwise products

$$q_{ijm}(r, \vartheta, \varphi) = (f_i \otimes Y_{jm})(r, \vartheta, \varphi) = f_i(r) Y_{jm}(\vartheta, \varphi) \tag{10}$$

in function space. Accordingly, tree s_k in a parser sequence Fig. 3 or Fig. 4 is represented by a linear combination

$$s_k(r, \vartheta, \varphi) = \mathrm{Re} \left[\sum_{ijm} a_{kijm} \, q_{ijm}(r, \vartheta, \varphi) \right] \tag{11}$$

of functions, where the a_{kijm} abbreviate the Clebsch-Gordan coefficients, saying that filler i is bound to a superposition of spherical harmonics Y_{jm} in tree k. Additionally, we take real parts in order to obtain real-valued representations.

Furthermore, we introduce an *observation model* (beim Graben et al. 2008a) for visualization purposes, by

$$E(\vartheta, \varphi) = \int_0^1 |s(r, \vartheta, \varphi)| \, \mathrm{d}r \tag{12}$$

Fig. 5. Sequence of functional tensor product representations (10) in the observation model (12) for the tree sequence Fig. 3.

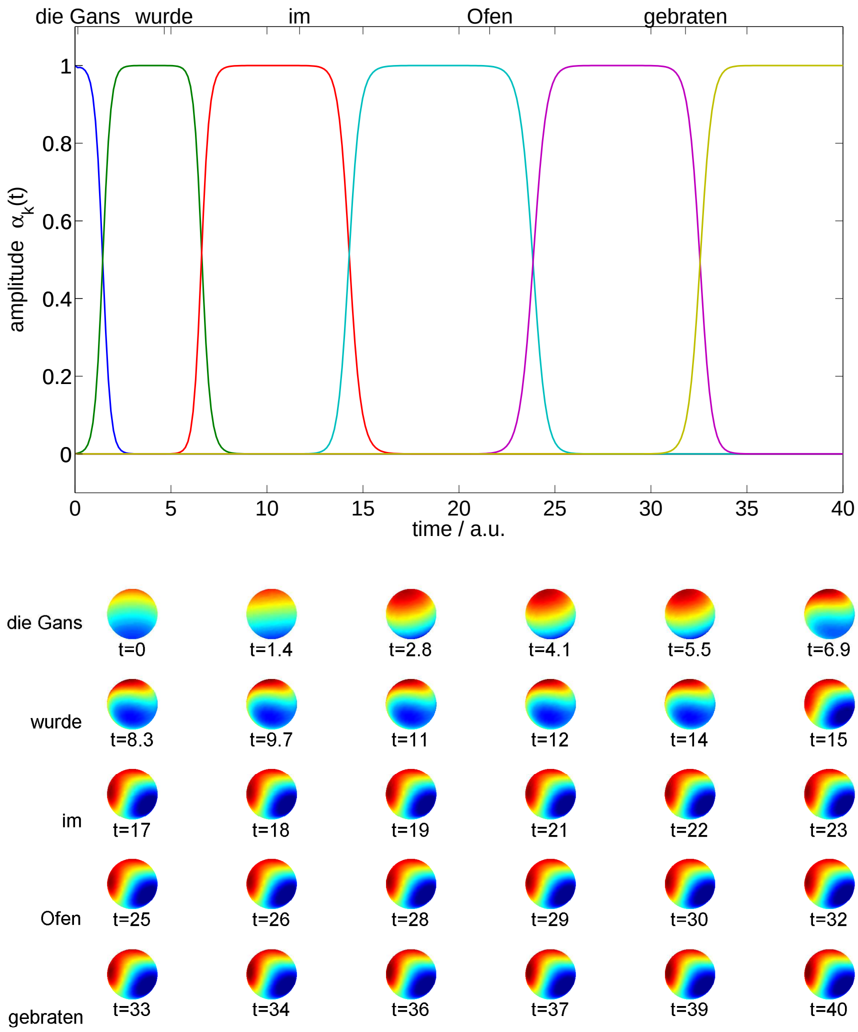
Next we encounter the second mapping problem, embedding the time-discrete tensor product representations (10) into continuous time. This is achieved by regarding the 7 different states s_k from both parses Figs. 3, 4 as saddle nodes in an SHS (Rabinovich et al. 2001, Afraimovich et al. 2004, Rabinovich et al. 2008). To this end, we introduce an order parameter expansion (Haken 1983)

$$u(r, \vartheta, \varphi, t) = \sum_{k=1}^{n} \alpha_k(t) s_k(r, \vartheta, \varphi) \tag{13}$$

where n is the total number of representation states and $\alpha_k(t) \in [0, 1]$ is the time-dependent activation of the kth state. These amplitudes are delivered by a winnerless competition in a generalized Lotka-Volterra system of $n = 7$ populations

$$\begin{aligned} \frac{\mathrm{d}\xi_k}{\mathrm{d}t} &= \xi_k \left(\sigma_k - \sum_{j=1}^{n} \rho_{kj} \xi_j \right) \\ \alpha_k(t) &= \frac{\xi_k}{\sigma_k} \end{aligned} \tag{14}$$

with growth rates $\sigma_k > 0$, and interaction weights $\rho_{kj} > 0$, $\rho_{kk} = 1$, separately trained by the algorithm of Afraimovich et al. (2004) and Rabinovich et al. (2008) for the sequence $(s_1, s_2, s_3, s_4, s_5, s_6)$ of filler states from Fig. 3 and for the sequence $(s_1, s_2, s_3, s_4, s_7)$ of violation states from Fig. 4. The result of this continuous time embedding is shown in Fig. 6 for the evolution of filler states.



(b)

Fig. 6. Continuous time embedding of tensor product representations from Fig. 5 through winnerless competition of activation [Eq. (14)] amplitudes [Eq. (13)]. (a) Sequential amplitude dynamics. (b) Snapshots of EEG model [Eq. (12)] dynamics for filler sentence (3) (color coded amplitude range: -30 to 30).

3.3 Dynamic Fields

In the third step of DCM, the constructed spatio-temporal dynamics $u_{\text{filler}}(x,t)$, $u_{\text{violation}}(x,t)$ for the parsing of filler sentences (3) and phrase structure violation sentences (2), respectively, can be used as training data for solving the *inverse problem* in dynamic field theory (Potthast and beim Graben 2009, beim Graben and Potthast 2009). Starting from the often deployed Amari equation (Amari 1977) for a neural field $u(x,t)$,

$$\tau \frac{\partial u(x,t)}{\partial t} + u(x,t) = \int_D w(x,y) f(u(y,t)) \, \mathrm{d}y \,, \tag{15}$$

with time constant τ, synaptic weight kernel $w(x,y)$, sigmoidal activation function $f(u)$ and feature space D, the inverse problem is posed by determining the kernel $w(x,y)$ from a prescribed trajectory $u(x,t)$. Here, $x = (r, \vartheta, \varphi) \in D = S$ denotes a point within the unit sphere. Going along the lines of Potthast and beim Graben (2009), we construct a biorthogonal function system $s_j^{\perp}(x)$ from the representational states (11), obeying

$$\int_D s_j^{\perp}(x) s_k(x) \, \mathrm{d}x = \delta_{jk} \,. \tag{16}$$

Then, we obtain from (13)

$$\begin{aligned}
\int_D s_j^{\perp}(x) u(x,t) \, \mathrm{d}x &= \sum_{k=1}^{n} \alpha_k(t) \int_D s_j^{\perp}(x) s_k(x) \, \mathrm{d}x \\
\int_D s_j^{\perp}(x) u(x,t) \, \mathrm{d}x &= \sum_{k=1}^{n} \alpha_k(t) \delta_{jk} \\
\alpha_j(t) &= \int_D s_j^{\perp}(x) u(x,t) \, \mathrm{d}x \\
\xi_j(t) &= \sigma_j \int_D s_j^{\perp}(x) u(x,t) \, \mathrm{d}x \,.
\end{aligned} \tag{17}$$

Next, we derivate (13) with respect to time t, by exploiting (14)

$$\begin{aligned}
\frac{\partial u(x,t)}{\partial t} &= \sum_k \frac{1}{\sigma_k} \frac{\mathrm{d}\xi_k}{\mathrm{d}t} s_k(x) \\
\frac{\partial u(x,t)}{\partial t} &= \sum_k \frac{1}{\sigma_k} \left[\xi_k \left(\sigma_k - \sum_j \rho_{kj} \xi_j \right) \right] s_k(x) \\
\frac{\partial u(x,t)}{\partial t} &= \sum_k \xi_k s_k(x) - \sum_{kj} \frac{\rho_{kj}}{\sigma_k} \xi_k \xi_j s_k(x) \,.
\end{aligned} \tag{18}$$

Multiplying with τ and adding (13), we obtain the left hand side of the Amari equation (15)

$$\tau \frac{\partial u(x,t)}{\partial t} + u(x,t) = \sum_k \tau \xi_k s_k(x) - \sum_{kj} \frac{\tau \rho_{kj}}{\sigma_k} \xi_k \xi_j s_k(x) + \sum_k \frac{1}{\sigma_k} \xi_k s_k(x)$$

$$\tau \frac{\partial u(x,t)}{\partial t} + u(x,t) = \sum_k \left(\tau + \frac{1}{\sigma_k} \right) \xi_k s_k(x) - \sum_{kj} \frac{\tau \rho_{kj}}{\sigma_k} \xi_k \xi_j s_k(x) \,. \tag{19}$$

Now we can eliminate all occurrences of ξ by means of (17) which gives

$$\begin{aligned}
\tau \frac{\partial u(x,t)}{\partial t} + u(x,t) &= \sum_k \left(\tau + \frac{1}{\sigma_k} \right) \sigma_k \int_D s_k^\perp(y) u(y,t) s_k(x) \,\mathrm{d}y - \\
&- \sum_{kj} \frac{\tau \rho_{kj}}{\sigma_k} \sigma_k \sigma_j \int_D \int_D s_k^\perp(y) u(y,t) s_j^\perp(z) u(z,t) s_k(x) \,\mathrm{d}y \,\mathrm{d}z \\
\tau \frac{\partial u(x,t)}{\partial t} + u(x,t) &= \int_D \sum_k (\tau \sigma_k + 1)\, s_k^\perp(y) s_k(x) u(y,t) \,\mathrm{d}y - \qquad (20) \\
&- \int_D \int_D \sum_{kj} \tau \rho_{kj} \sigma_j s_k^\perp(y) s_j^\perp(z) s_k(x) u(y,t) u(z,t) \,\mathrm{d}y \,\mathrm{d}z \,.
\end{aligned}$$

In the next step we consider the right hand side of the Amari equation (15) which describes a nonlinear integral transformation in space

$$v_t(x) = \mathcal{J}[u(\cdot,t)](x) = \int_D w(x,y) f(u(y,t)) \,\mathrm{d}y \,, \tag{21}$$

when fixing the time point t. The transformation (21) can be expressed as a functional Taylor expansion (sometimes called *Volterra series* when applied to the time domain),

$$v_t(x) = w_0(x) + \sum_{m=1}^{\infty} \frac{1}{m!} \int_D \cdots \int_D w_m(x, y_1, y_2, \ldots, y_m) u(y_1,t) u(y_2,t) \ldots u(y_m,t) \,\mathrm{d}y_1 \,\mathrm{d}y_2 \cdots \mathrm{d}y_m . \tag{22}$$

Taking only the first three terms into account yields

$$\begin{aligned}
&\tau \frac{\partial u(x,t)}{\partial t} + u(x,t) \qquad (23) \\
&= w_0 + \int_D w_1(x,y) u(y,t) \,\mathrm{d}y + \frac{1}{2} \int_D \int_D w_2(x,y,z) u(y,t) u(z,t) \,\mathrm{d}y \,\mathrm{d}z \,.
\end{aligned}$$

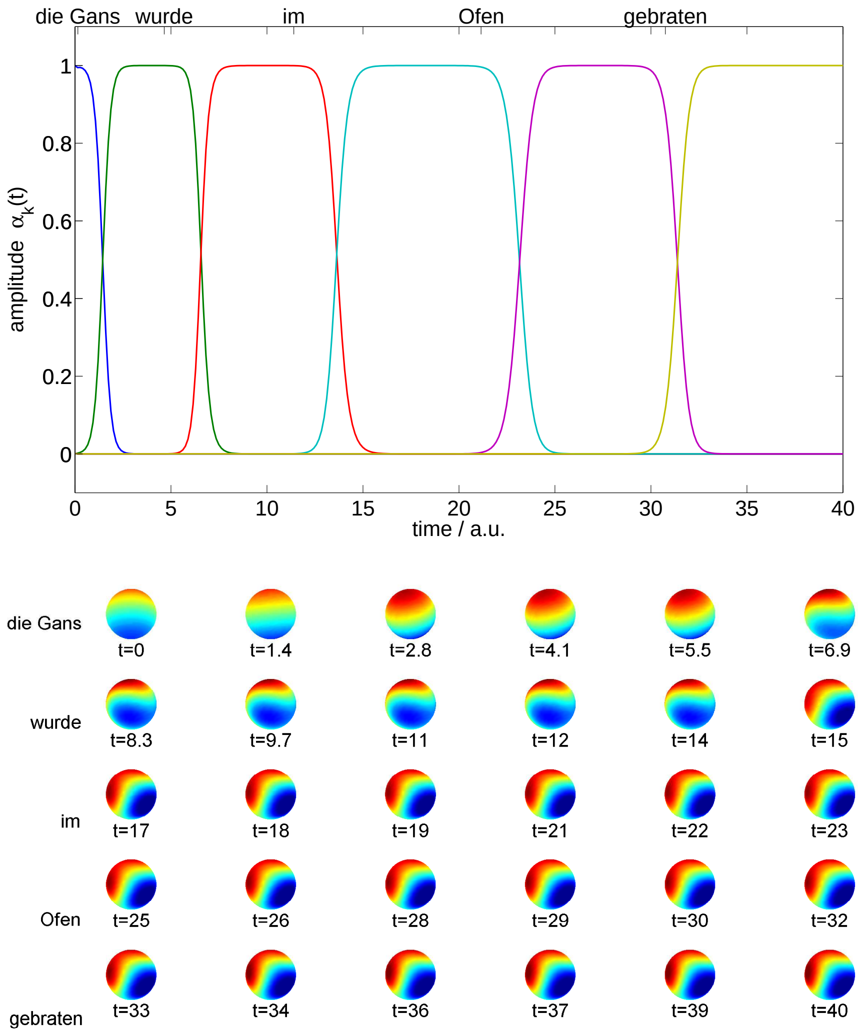
Finally we compare (20) and (23) to obtain the kernels

$$\begin{aligned}
w_0 &= 0 \qquad (24) \\
w_1(x,y) &= \sum_k (\tau \sigma_k + 1) s_k^\perp(y) s_k(x) \\
w_2(x,y,z) &= 2\tau \sum_{kj} \rho_{kj} \sigma_j s_k^\perp(y) s_j^\perp(z) s_k(x) \,.
\end{aligned}$$

Basically, kernel $w_1(x, y)$ describes a Hebbian synapse between sites y and x that have been trained with pattern sequence s_k. This finding confirms the previous result of Potthast and beim Graben (2009). Besides the three-point kernel $w_2(x, y, z)$ further generalizes Hebbian learning to interactions between three sites x, y, z in feature space. Hence we explicitly solved the inverse problem for dynamic field theory with winnerless competition.

4 Results

First we present the dynamics of the Amari field equation (15) with kernels (24) constructed for the parse of the filler sentence (3) in Fig. 7.



(b)

Fig. 7. Solutions of the Amari equation (15) with kernels (24) constructed for the parse of the filler sentence (3). (a) Sequential amplitude dynamics. (b) Snapshots of EEG model [Eq. (12)] dynamics for filler sentence (3) (color coded amplitude range: −30 to 30).

Comparison with Fig. 6 reveals some slight phase shifts in the amplitude dynamics that are due to the numerical instabilty of the Lotka-Volterra dynamics (14). Nevertheless, the simulated data are in considerably good agreement with the training data.

Finally, we present the EEG observation model of the differences $u_{\text{violation}}(x,t) - u_{\text{filler}}(x,t)$ in Fig. 8.

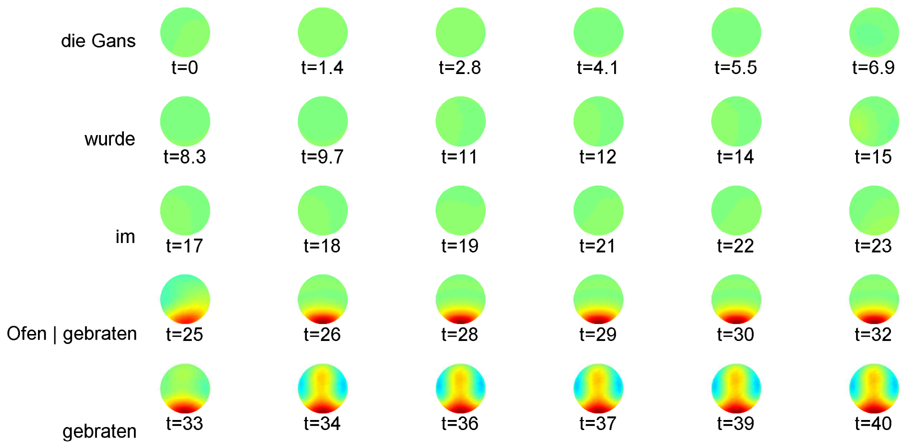


Fig. 8. Snapshot of the temporal evolution of the model EEG differences [Eq. (12)] between dynamic field patterns $u_{\text{filler}}(x,t)$, $u_{\text{violation}}(x,t)$. The difference elicited by the prediction failure in condition (2) compared to condition (3) resembles the observed P600 ERP in Fig. 1 (color coded amplitude range: -5 to 5).

There are no differences between conditions (2) and (3) until *gebraten* ("grilled") was presented to the parser in the violation condition (2) when the parser expects a noun *Ofen* ("oven") immediately following the preposition *im* ("in the"). In fact, this happens in the filler condition (3), whereas the verb *gebraten* in condition (2) leads to a prediction failure, symbolically indicated by $\perp$ in Fig. 4. Here, the parse breaks down, eliciting a P600 component in the ERP, shown in Fig. 1. Interestingly, our dynamic field model replicates this effect, at least phenomenologically, as a positivity with a distinct, parietally focussed topography.

5 Conclusion

In this contribution we presented a phenomenological account to dynamic field models for language-related brain potentials. For a paradigmatic ERP experiment on the processing of phrase structure violations in German (Hahne and Friederici 1999), we firstly derived a context-free grammar representing the stimulus material. Then, we described the phrase structure violation by a prediction error in an interactive left-corner parser. Using filler/role decompositions, tensor product representations and winnerless competition (Mizraji 1989, Smolensky 1990, Smolensky and Legendre 2006, Smolensky 2006, Rabinovich et al. 2001, Afraimovich et al. 2004, Rabinovich et al. 2008) in dynamic cognitive modeling (DCM: beim Graben and Potthast (2009)), we constructed dynamic fields over compact feature spaces (Erlhagen and Schöner 2002, Schöner and Thelen 2006) in order to represent the cognitive computations by neurodynamic systems. We solved the inverse problem for synaptic weight kernel construction for the Amari neural field equation (Amari 1977, Potthast and beim Graben 2009, beim Graben and Potthast 2009). Finally, we replicated the experimentally observed P600 ERP component elicited by the phrase structure violation in our phenomenological DCM model. In the current state, DCM establishes a framework to solve neurodynamic inverse problems in a top-down fashion. It does not yet address another inverse problem prevalent in the cognitive neurosciences, namely the bottom-up reconstruction of neurodynamics from observed physiological time series, such as EEG or ERP data (Regan 1972, beim Graben et al. 2000, Makeig et al. 2002; 2004). In particular, it is not related to dynamic causal modeling (David and Friston 2003, David et al. 2006) addressing the inverse problem of finding neural generators from physiological data. Certainly, dynamic causal modeling and dynamic cognitive modeling ought to be connected at the intermediate level of neurodynamics. We hope that dynamic cognitive modeling and dynamic causal modeling could be eventually unified.

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