Quantum Representation Theory for Nonlinear Dynamical Automata

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To appear in R. Wang & F. Gu (Eds.): *Proceedings of the First International Conference on Cognitive Neurodynamics*, Springer, 2008.

Abstract. Nonlinear dynamical automata (NDAs) are implementations of Turing machines by nonlinear dynamical systems. In order to use them as parsers, the whole string to be processed has to be encoded in the initial conditions of the dynamics. This is, however, rather unnatural for modeling human language processing. I shall outline an extension of NDAs that is able to cope with that problem. The idea is to encode only a "working memory" by a set of initial conditions in the system's phase space, while incoming new material then acts like "quantum operators" upon the phase space thus mapping a set of initial conditions onto another set. Because strings can be concatenated non-commutatively, they form the word semigroup, whose algebraic properties must be preserved by this mapping. This leads to an algebraic representation theory of the word semigroup by quantum operators acting upon the phase space of the NDA.

1 Introduction

One of the most crucial problems in cognitive neurodynamics is the realization of symbolic processing capabilities in the human brain. While the traditional cognitivistic account assumes that cognition is essentially rule-driven manipulation of symbols [1, 2], the dynamical systems approach to cognition models cognitive processes by the (transient) dynamics of nonlinear systems, such as neural networks [2, 3] or nonlinear dynamical automata (NDA) [2, 4, 5]. The latter are piecewise affine linear maps on the unit square whose symbolic dynamics exhibits the computational power of Turing machines.

Therefore, NDAs have been suggested for modeling syntactic language processing [2, 5] and language-related brain potentials (ERPs) [6]. It is the aim of this paper to discuss one particular problem of this approach, the issue of nondeterminacy of parsing [7], and to present a solution in terms of interactive computation [8] such that environmental perturbations act upon the system's phase state similar to quantum operators.

2 Nonlinear Dynamical Automata

In this section, we construct an NDA that is able to process the material from the study of Osterhout et al. [6] investigating sentences such as "the judge believed the defendant was lying". In a first step, the sentence is described by a phrase structure tree as in Fig. 1.

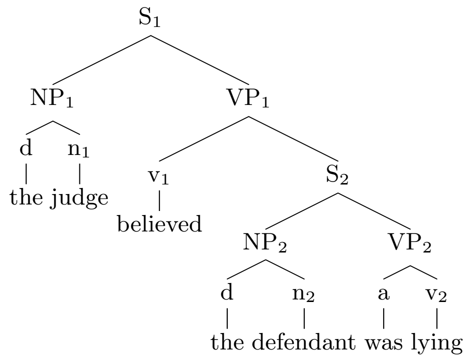


Fig. 1. Phrase structure tree of the example sentence.

In order to derive a context-free grammar (CFG) from this tree, we first discard the lexical material, regarding the nodes $\mathrm{d}, \mathrm{n}_1, \mathrm{v}_1, \mathrm{n}_2, \mathrm{a}$, and v_2 as terminal symbols. The CFG is then given by the rules: (1) $\mathrm{S}_1 \to \mathrm{NP}_1\,\mathrm{VP}_1$, (2) $\mathrm{NP}_1 \to \mathrm{d}\,\mathrm{n}_1$, (3) $\mathrm{VP}_1 \to \mathrm{v}_1\,\mathrm{S}_2$, (4) $\mathrm{S}_2 \to \mathrm{NP}_2\,\mathrm{VP}_2$, (5) $\mathrm{NP}_2 \to \mathrm{d}\,\mathrm{n}_2$, and (6) $\mathrm{VP}_2 \to \mathrm{a}\,\mathrm{v}_2$. Following [5], the variables of that CFG are next Gödel encoded by integers from a g-adic number system. Let $G(\mathrm{d}) = 0, G(\mathrm{n}_1) = 1, G(\mathrm{v}_1) = 2, G(\mathrm{n}_2) = 3, G(\mathrm{a}) = 4, G(\mathrm{v}_2) = 5, G(\mathrm{NP}_1) = 6, G(\mathrm{NP}_2) = 7, G(\mathrm{VP}_1) = 8, G(\mathrm{VP}_2) = 9, G(\mathrm{S}_1) = \mathrm{A}$, and $G(\mathrm{S}_2) = \mathrm{B}$, where $\mathrm{A} \equiv 10, \mathrm{B} \equiv 11$. Then, the sentence "the judge believed the defendant was lying" is mapped onto the string 0120345, being interpreted as a fraction 0.0120345_6 where $g_T = 6$ is the number of terminal symbols. The total number of variables is $g_V = 12$.

Syntactic language processing (parsing) generally describes the mapping of a sentence to its phrase structure tree, such as in Fig. 1. Yet, the most simple parsers for CFGs are push-down automata which merely decide whether an input string can be generated by a grammar. In the following, we shall discuss a simple top-down parser. Its state descriptions comprise the input string w and a stack memory γ. Table 1 gives the sequence of state descriptions of a parser processing the string 0120345 according to the Gödel encoding of the CFG.

In order to map the state descriptions in Table 1 onto the dynamics on the NDA, we have to compute the Gödel numbers

$$G_{V|T}(x) = \sum_{i=1}^{|x|} G(x_i) g_{V|T}^{-i} + \sum_{i=|x|+1}^{\infty} \eta_i g_{V|T}^{-i} \quad (1)$$

Table 1. Sequence of state descriptions of a top-down parser processing the string 0120345. The operations are indicated as follows: `predict (X)` means prediction according to rule (X) in the CFG; `attach` means cancelation of successfully predicted terminals both from stack and input; and `accept` means acceptance of the string as being well-formed.

time	stack γ	input w	operation
0	A	0120345	`predict (1)`
1	68	0120345	`predict (2)`
2	018	0120345	`attach`
3	18	120345	`attach`
4	8	20345	`predict (3)`
5	2B	20345	`attach`
6	B	0345	`predict (4)`
7	79	0345	`predict (5)`
8	039	0345	`attach`
9	39	345	`attach`
10	9	45	`predict (6)`
11	45	45	`attach`
12	5	5	`attach`
13	ε	ε	`accept`

in the bases g_V, g_T, respectively. Here, $|x|$ denotes the lengths of the strings $x = \gamma$ or w and η_i are random digits in the respective base. By virtue of (1), the parser's state descriptions are represented by clouds of points $(G_V(\gamma), G_T(w))$ randomly scattered across rectangles in the unit square $X = [0, 1]^2$. Correspondingly, Table 1 is represented by a sequence of those rectangular macrostates evolving due to a nonlinear map $\Phi : X \to X$ [2, 4, 5]. Figure 2(a) displays the parsing trajectory processing the string 0120345 according to Table 1.

3 Quantum Representation Theory

The initial conditions of the NDA model are given by the whole string $w =$ 0120345 in the input. However, this is cognitively implausible for two reasons: 1) Hearing or reading supplies lexical material successively to the human mind and not at once as modeled above. The same holds for psycholinguistic experiments with a word-by-word presentation paradigm [6]. 2) The NDA's dynamics is deterministic for a given initial condition. Hence, the preparation of initial conditions by the whole string w, yields a completely predictable trajectory in phase space. By contrast, the human parser is often trapped by garden path interpretations resulting from unpredicted continuations [6, 7].

In order to remedy this shortcoming, I suggest the following solution: 1) Restrict the parser's input to a *working memory* of finite length, say, $w = w_1 w_2$, $|w| = 2$ [7]. 2) After each attachment (i.e. when $w = w_2$) the next symbol A is scanned from an *information source* [9]. 3) Define a map $\mathtt{scan}(A) : w \mapsto w'$, such that $w' = w_2 A$. A parser described in this way, is not longer a closed system. Now, it is interacting with its environment which perturbs its state descriptions [8]. What does this mean for the corresponding NDA?

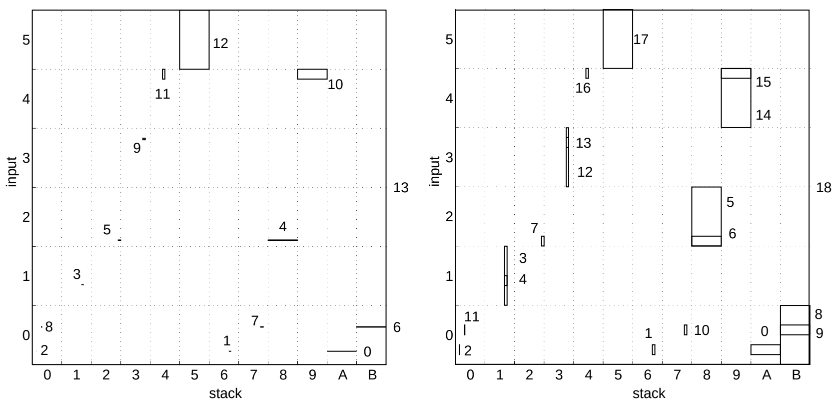
Since the NDA is characterized by a nonlinear dynamics $\Phi : X \to X$, the scanned input A must be *represented* by another map $\rho(A) : X \to X$. Such representations are well-known in algebraic quantum theory [10], where observables $A, B \in \mathcal{A}$ are described by their abstract commutation relations. A representation is then a C*-algebra homomorphism $\rho : \mathcal{A} \to \mathfrak{B}(\mathcal{H})$ from a C*-algebra $\mathcal{A}$ into the set of bounded operators $\mathfrak{B}(\mathcal{H})$ acting upon a Hilbert space $\mathcal{H}$.

In terms of quantum representation theory, the construction of the representation $\rho(A)$ in the NDA's phase space is straightforward. After attaching w_1, the working memory only contains the most significant digit $w = w_2$, thus, by (1), possessing the Gödel code $G_T(w) = G(w_2)g_T^{-1} + \sum_{i=2}^{\infty} \eta_i g_T^{-i}$. Inserting A into the second most significant position by the `scan` operation, yields $w' = w_2 A$. Therefore, the representation $\rho(A)$ is given by

$$\rho(A)[(G_V(\gamma), G_T(w))] = (G_V(\gamma), G_T(w'))$$

$$G_T(w') = G(w_2)g_T^{-1} + G(A)g_T^{-2} + \sum_{i=3}^{\infty} \eta_i g_T^{-i} \,. \qquad (2)$$

Supplementing the top-down parser from Sect. 2 with the `scan` operation and representing it by (2) in the NDA's phase space, the parsing trajectory in Fig. 2(b) is obtained. After each attachment, the macrostate coincides with one of those rectangles partitioning the unit square. Immediately afterwards, `scan` fetches the next symbol from the information source that acts like a quantum operator upon the unit square thereby squeezing the rectangular macrostate vertically.



(a) (b)

Fig. 2. Parsing trajectories for the string $w = 0120345$. (a) Fully encoded in the initial conditions. The indices correspond to the time steps in Table 1. (b) Only working memory encoded in the initial conditions and representing the `scan` operation.

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